Attendance Problems. Evaluate the following

1. $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

2. $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

3. $\dfrac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2}$

4. $\dfrac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2}$

5. $\dfrac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(3 \cdot 2 \cdot 1)}$

6. $\dfrac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)}$

- I can solve problems involving the Fundamental Counting Principle.
- I can solve problems involving permutations and combinations.

Vocabulary	
Fundamental Counting Principle	permutation
factorial	combination

Common Core: CC.9-12.S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

You have previously used tree diagrams to find the number of possible combinations of a group of objects. In this lesson, you will learn to use the **Fundamental Counting Principle**.

Fundamental Counting Principle

If there are n items and m_1 ways to choose a first item, m_2 ways to choose a second item after the first item has been chosen, and so on, then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to choose n items.

1 Using the Fundamental Counting Principle

A **For the lunch special, you can choose an entrée, a drink, and one side dish. How many meal choices are there?**

number of main dishes	times	number of beverages	times	number of sides	equals	number of choices
3	$\times$	4	$\times$	3	$=$	36

There are 36 meal choices.

B **In Utah, a license plate consists of 3 digits followed by 3 letters. The letters *I*, *O*, and *Q* are not used, and each digit or letter may be used more than once. How many different license plates are possible?**

digit		*digit*		*digit*		*letter*		*letter*		*letter*	
10	$\times$	10	$\times$	10	$\times$	23	$\times$	23	$\times$	23	$= 12{,}167{,}000$

There are 12,167,000 possible license plates.

"A man is a success if he gets up in the morning and gets to bed at night and in between does what he wants to do."—*Songwriter, Bob Dylan*

Example 1.

A. To make a yogurt parfait, you choose one flavor of yogurt, one fruit topping, and one nut topping. How many parfait choices are there?

Yogurt Parfait (choose 1 of each)		
Flavor	**Fruit**	**Nuts**
Plain	Peaches	Almonds
Vanilla	Strawberries	Peanuts
	Bananas	Walnuts
	Raspberries	
	Blueberries	

B. A password for a site consists of 4 digits followed by 2 letters. The letters *A* and *Z* are not used, and each digit or letter many be used more than once. How many unique passwords are possible?

Guided Practice.

11. A "make-your-own-adventure" story lets you choose 6 starting points, gives 4 plot choices, and then has 5 possible endings. How many adventures are there?

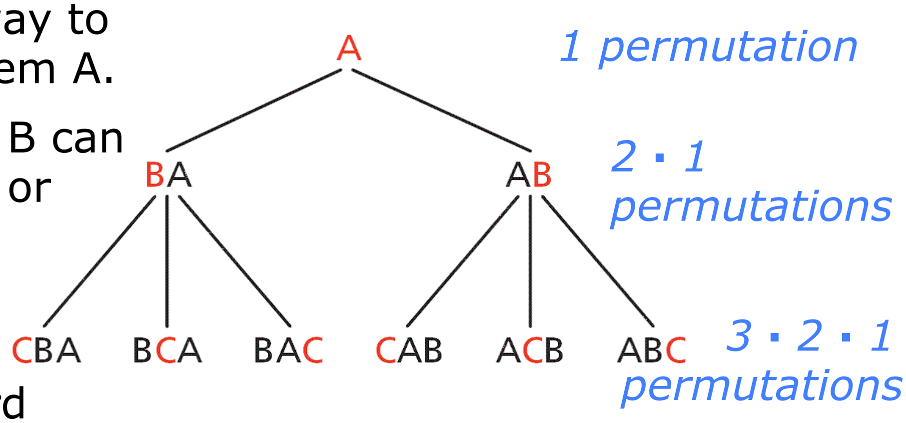
12. A "weak" password consists of consists of 8 lower case letters. A strong password consists of the same 8, but includes upper and lower case letters, numbers and symbols. What is the difference between the possible number of passwords between strong and weak password?

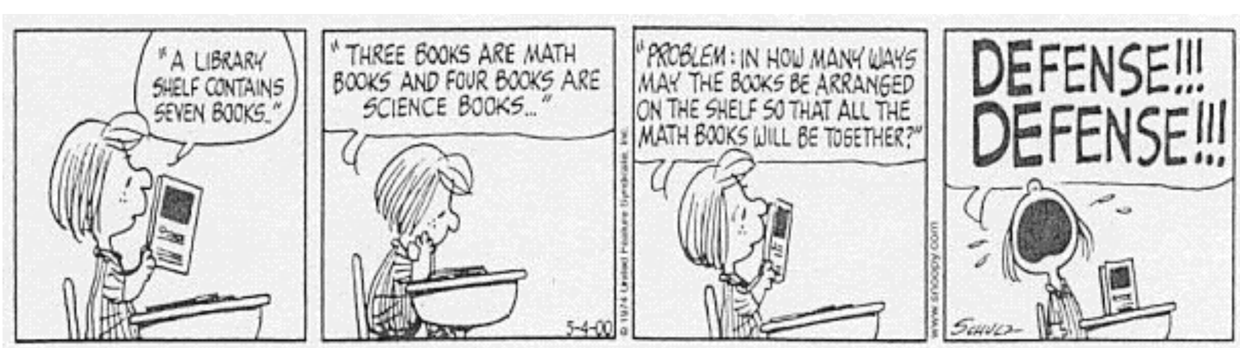
A **permutation** is a selection of a group of objects in which order is important.

There is one way to arrange one item A.

A second item B can be placed first or second.

A third item C can be first, second, or third for each order above.





You can see that the number of permutations of 3 items is $3 \cdot 2 \cdot 1$. You can extend this to permutations of n items, which is $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 1$. This expression is called *n factorial*, and is written as $n!$.

n Factorial

For any whole number n,

WORDS	NUMBERS	ALGEBRA
The **factorial** of a number is the product of the natural numbers less than or equal to the number. 0! is defined as 1.	$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$	$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 1$

Sometimes you may not want to order an entire set of items. Suppose that you want to select and order 3 people from a group of 7. One way to find possible permutations is to use the Fundamental Counting Principle.

First Person		Second Person		Third Person		*There are 7 people. You are choosing 3 of them in order.*
7 choices	×	6 choices	×	5 choices	=	210 permutations

Q: How can you tell when a factorial is enthusiastic?
A: It's always enthusiastic—it has an exclamation point!

Another way to find the possible permutations is to use factorials. You can divide the total number of arrangements by the number of arrangements that are not used. In the previous slide, there are 7 total people and 4 whose arrangements do not matter.

$$\frac{\text{arrangements of 7}}{\text{arrangements of 4}} = \frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 210$$

Permutations

NUMBERS	ALGEBRA
The number of permutations of 7 items taken 3 at a time is $${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!}.$$	The number of permutations of n items taken r at a time is $${}_nP_r = \frac{n!}{(n-r)!}.$$

2 Finding Permutations

A How many ways can a club select a president, a vice president, and a secretary from a group of 5 people?

This is the equivalent of selecting and arranging 3 items from 5.

$$_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!}$$

Substitute 5 for n and 3 for r in $\frac{n!}{(n-r)!}$.

$$= \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2 \cdot 1}}{\cancel{2 \cdot 1}}$$

Divide out common factors.

$$= 5 \cdot 4 \cdot 3 = 60$$

There are 60 ways to select the 3 people.

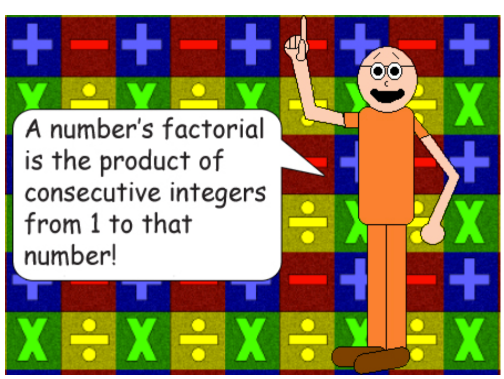
B An art gallery has 9 fine-art photographs from an artist and will display 4 from left to right along a wall. In how many ways can the gallery select and display the 4 photographs?

$$_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}$$

Divide out common factors.

$$= 9 \cdot 8 \cdot 7 \cdot 6$$
$$= 3024$$

There are 3024 ways that the gallery can select and display the photographs.



Example 2

A. How many ways can a student government select a president, vice president, secretary, and treasurer from a group of 6 people?

B. How many ways can a stylist arrange 5 of 8 vases from left to right in a store display?

Guided Practice.

13. How many ways can a 2-digit number be formed by using only the digits 5–9 and by each digit being used only once?

14. Awards are given out at a costume party. How many ways can "most creative," "silliest," and "best" costume be awarded to 8 contestants if no one gets more than one award?

A **combination** is a grouping of items in which order does not matter. There are generally fewer ways to select items when order does not matter. For example, there are 6 ways to order 3 items, but they are all the same combination:

6 permutations ➔{ABC, ACB, BAC, BCA, CAB, CBA}

1 combination ➔ {ABC}

To find the number of combinations, the formula for permutations can be modified.

$$\text{number of permutations} = \frac{\text{ways to arrange all items}}{\text{ways to arrange items not selected}}$$

Because order does not matter, divide the number of permutations by the number of ways to arrange the selected items.

$$\text{number of combinations} = \frac{\text{ways to arrange all items}}{(\text{ways to arrange selected items})(\text{ways to arrange items not selected})}$$

Combinations

NUMBERS	ALGEBRA
The number of combinations of 7 items taken 3 at a time is $_7C_3 = \frac{7!}{3!(7-3)!}$.	The number of combinations of n items taken r at a time is $_nC_r = \frac{n!}{r!(n-r)!}$.

When deciding whether to use permutations or combinations, first decide whether order is important. Use a permutation if order matters and a combination if order does not matter.

Helpful Hint

You can find permutations and combinations by using **nPr** and **nCr**, respectively, on scientific and graphing calculators.

3 *Pet Adoption Application*

Katie is going to adopt kittens from a litter of 11. How many ways can she choose a group of 3 kittens?

Step 1 Determine whether the problem represents a permutation or combination.

The order does not matter. The group Kitty, Smoky, and Tigger is the same as Tigger, Kitty, and Smoky. It is a combination.

Step 2 Use the formula for combinations.

$${}_{11}C_3 = \frac{11!}{3!(11-3)!} = \frac{11!}{3!(8!)} \quad \textit{n = 11 and r = 3}$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot \cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{3 \cdot 2 \cdot 1(\cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1})} \quad \textit{Divide out common factors.}$$

$$= \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = \frac{11 \cdot \cancel{10}^{5} \cdot \cancel{9}^{3}}{{}_{1}\cancel{3} \cdot {}_{1}\cancel{2} \cdot 1} = 165$$

There are 165 ways to select a group of 3 kittens from 11.

Example 3. There are 12 different-colored cubes in a bag. How many ways can Randall draw a set of 4 cubes from the bag?

15. <u>Guided Practice</u>. The swim team has 8 swimmers. Two swimmers will be selected to swim in the first heat. How many ways can the swimmers be selected?

16. A powerball lottery ticket consists of 5 white numbers from 1-59 and "powerball" from 1 of 35 numbers. How many different powerball lottery tickets are possible?

13-1 Permutations and Combinations (p 874) 9, 10, 11, 13, 14, 24, 25, 28-31, 36, 37, 38.